Thermal lensing in cryogenic sapphire substrates

Takayuki Tomaru$^{1,3}$, Toshikazu Suzuki$^2$, Shinji Miyoki$^1$, Takashi Uchiyama$^2$, C T Taylor$^1$, Akira Yamamoto$^2$, Takakazu Shintomi$^2$, Masatake Ohashi$^1$ and Kazuaki Kuroda$^1$

$^1$ Institute for Cosmic Ray Research (ICRR), The University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, Chiba 277-8582, Japan
$^2$ High Energy Accelerator Research Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan
E-mail: tomaru@post.kek.jp

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Abstract
We report the reduction of the thermal lensing in cryogenic sapphire mirrors, which is planned to be used in the large scale cryogenic gravitational wave telescope project. We measured three key parameters of sapphire substrate for thermal lensing at cryogenic temperature. They are optical absorption coefficient, thermal conductivity and temperature coefficient of refractive index at cryogenic temperature. On the basis of these measurements, we estimated the shot noise sensitivity of the interferometer with thermal lensing by using a wavefront tracing simulation. We found that thermal lensing in cryogenic sapphire mirrors is negligible.

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1. Introduction

Thermal lensing is a serious problem for the next generation interferometric gravitational wave detectors. Thermal lensing is an effect of wavefront distortion of laser beam caused by refractive index distribution in the mirrors with temperature distribution due to optical absorption. It causes mode mismatching between beam wavefront and mirror surface, and finally increases the photon shot noise [1]. Although mono-crystalline sapphire mirror substrate is one of the most effective materials to reduce thermal noise [2–4], it has large optical absorption at the present stage. Large efforts to reduce thermal lensing by reducing optical absorption in sapphire have been made by some groups [5–7]. In this paper, we report on the reduction of the thermal lensing in the cryogenic sapphire mirrors, which are planned for use.
in the large scale cryogenic gravitational wave telescope (LCGT) project [8] to reduce thermal noise drastically [9, 10]. Firstly, we describe the measurement of key parameters of sapphire substrate at cryogenic temperature, which are needed to estimate wavefront distortion caused by thermal lensing. Thereafter, we discuss the shot noise sensitivity with thermal lensing by using a wavefront tracing simulation (an FFT simulation) [11].

2. Measurement of key parameters of sapphire substrate for thermal lensing at cryogenic temperature

The sagitta of wavefront distortion of laser beam $ds$ caused by the thermal lensing depends on three parameters of mirror substrate,

$$ds \propto \frac{\alpha \beta}{\kappa}$$

where $\alpha$ is the optical absorption coefficient in the mirror, $\kappa$ thermal conductivity of the mirror substrate and $\beta$ temperature coefficient of the refractive index ($dn/dT$) of the mirror substrate.

Firstly, we measured optical absorption coefficient in cryogenic sapphire by laser calorimetry method [12]. The sample temperature was 5 K and light wavelength was 1.064 $\mu$m (Nd:YAG laser). We measured two sapphire samples, ‘CSI white high purity’ and ‘Hemlite’, both manufactured by Crystal Systems Inc. [13]. The dimensions of ‘CSI white’ are 10 mm in diameter and 150 mm in thickness, and those of ‘Hemlite’ are 100 mm in diameter and 60 mm in thickness. The results show about 90 ppm cm$^{-1}$ optical absorption for both samples [14]. Also, there is no large difference of optical absorption coefficient at different positions in a sample for both samples. At room temperature, there are several reports of optical absorption in small sapphire samples by the photo-thermal method [5–7, 15]. Although their values lie in a range from 3 ppm cm$^{-1}$ to 550 ppm cm$^{-1}$, typical values are between 40 ppm cm$^{-1}$ and 100 ppm cm$^{-1}$.

Secondly, we measured thermal conductivity of sapphire in the range from 4 K to 40 K. Thermal conductivity of crystals at cryogenic temperature largely depends on sample quality. The ‘CSI white’ sample, used in optical absorption measurement, was used. The method for measurement is the longitudinal heat flow method [16]. The result shows that the maximum thermal conductivity of the sample was $4.3 \times 10^3$ W m$^{-1}$ K$^{-1}$ at 20 K. This value is about four times smaller than the value in the data book [16]. However, this value is two orders of magnitude larger than the value obtained at room temperature.

Lastly, we measured the temperature coefficient of refractive index $\beta$ of cryogenic sapphire by measuring the change in the refractive angle of light due to change in sample temperature. The displacement of the transmitted beam $\delta x$ caused by change in refractive index is described as

$$\delta x = \sin \left( \frac{\pi}{2} - \theta \right) \frac{d}{\cos \phi \cos \phi} \frac{1}{\tan \phi} \frac{\delta n}{n}$$

where $d$ is the sample length, $\phi$ the refractive angle, $n$ the refractive index of the sample at temperature $T$ and $\delta n$ the change in the refractive index. Injection beam angle, $\theta$, of 60° was used, which gave maximum displacement of the transmitted beam for sapphire. The ‘Hemlite’ sample, 100 mm in diameter and 60 mm in thickness, was used. To investigate thermal deformation of the sample, the displacement of the reflected beam from the sample was also monitored. Although the measured result was limited by thermal deformation of the sample, the upper limit of $|\beta| \leq 9 \times 10^{-8}$ K$^{-1}$ was obtained at 1.064 $\mu$m in an average between 5 K and 40 K. This value was two orders of magnitude smaller than that obtained at room temperature [17].
3. Thermal lensing in interferometer

Table 1 lists the thermal lensing parameters for fused silica and sapphire. This table shows that thermal lensing in cryogenic sapphire mirrors is at least four orders of magnitude smaller than that in both fused silica and sapphire at room temperature. We can regard the thermal lensing in fused silica at room temperature as almost the same as that in sapphire at room temperature.

To estimate the shot noise sensitivity of interferometers affected by thermal lensing, an FFT simulator is useful. The FFT simulator was developed by LIGO [11]. We assumed optical absorption in sapphire substrate of 90 ppm cm\(^{-1}\) and that in coating of 1 ppm. The LCGT design, the configuration being a power-recycled Fabry–Perot Michelson interferometer (figure 1), the finesse 100, the recycling gain 50 and the injection laser power 100 W, were used in this calculation. The phase maps of wavefront distortion caused by thermal lensing were calculated by Hello’s method [18] (figure 2 (a)). In the case of cryogenic mirrors, since thermal radiation from the mirrors is very small and they are cooled only by thermal conduction of suspension fibres, we calculated the phase maps by changing from the boundary condition of Hello’s calculation, which is the thermal radiation, to that of thermal conduction of a fibre (figure 2 (b)), where we assumed an axi-symmetric model of thermal conduction as the first approximation. Since the calculation of thermal lensing in the beam splitter has difficulty in axi-asymmetric temperature distribution, we took this effect into the simulation.
Figure 2. A calculation model of wavefront distortion caused by thermal lensing. (a) Room temperature case (Hello’s model). Thermal equilibrium in the mirror is achieved by optical absorption and thermal radiation [18]. (b) Cryogenic temperature case. A temperature of 20 K was assumed in the LCGT design. Thermal equilibrium in the mirror is achieved by optical absorption and thermal conduction of a fibre. As the first approximation, axi-symmetric heat flow was assumed.

Figure 3. Calculation results of the shot noise sensitivity with and without thermal lensing in sapphire mirrors at both room and cryogenic temperatures. ‘Ideal’ means the shot noise for the ideal interferometer. The values of ‘±0%’, ‘±30%’ and ‘±50%’ mean asymmetric optical absorption between both the near mirrors.

by assuming different thermal lensings between both arms. No compensative technique for wavefront distortion [19, 20, 21] was considered in this calculation.

Figure 3 shows the results of the shot noise sensitivity calculated by using the FFT simulation. This result shows that the shot noise sensitivity in the case of sapphire mirrors used at room temperature is two times worse than that in the ideal case due to thermal lensing. This sensitivity decline resulted from the reduction of recycling gain, from 50 to 16. In the
case where there are different optical absorptions between both the arms, decline of contrast also reduces sensitivity. On the other hand, thermal lensing in cryogenic sapphire mirrors is negligible, even in the case of ±50% asymmetric optical absorption between both the near mirrors. The cryogenic sapphire mirror effectively prevents thermal lensing.

4. Summary

We estimated thermal lensing in cryogenic sapphire mirrors on the basis of measurements. We conclude that the cryogenic sapphire mirror is effective not only for reducing thermal noise but also for suppressing thermal lensing. A remaining problem of cryogenic interferometers is a cooling problem due to large optical absorption in sapphire substrate, which will be solved in the near future.

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See endnote 1

See endnote 2
Endnotes

(1) Author: Kindly check reference [16]. Is there an author or editor to be added?
(2) Author: Please provide publication details (company and location) for both references in [23].

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